

**Cavitation Modeling in Euler and Navier-Stokes Codes**

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P-25

Many previous researchers have modeled sheet cavitation by means of a constant pressure solution in the cavity region coupled with a velocity potential formulation for the outer flow. The present paper discusses the issues involved in extending these cavitation models to Euler or Navier-Stokes codes. The approach taken is to start from a velocity potential model to ensure our results are compatible with those of previous researchers and available experimental data, and then to implement this model in both Euler and Navier-Stokes codes. The model is then augmented in the Navier-Stokes code by the inclusion of the energy equation which allows the effect of subcooling in the vicinity of the cavity interface to be modeled to take into account the experimentally observed reduction in cavity pressures that occurs in cryogenic fluids such as liquid hydrogen. Although our goal is to assess the practicality of implementing these cavitation models in existing three-dimensional, turbomachinery codes, the emphasis in the present paper will center on two-dimensional computations, most specifically isolated airfoils and cascades. Comparisons between velocity potential, Euler and Navier-Stokes implementations indicate they all produce consistent predictions. Comparisons with experimental results also indicate that the predictions are qualitatively correct and give a reasonable first estimate of sheet cavitation effects in both cryogenic and non-cryogenic fluids. The impact on CPU time and the code modifications required suggests that these models are appropriate for incorporation in current generation turbomachinery codes.

# Computational Modeling of Cavitation

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# Objectives

- Extend Existing Cavitation Theories from Potential Flow to Euler/Navier-Stokes Solvers
- Identify Techniques Needed to Incorporate Cavitation Models in Existing Turbomachinery Codes
- Add Additional Physics to Cavitation Model as Euler/Navier-Stokes Formulation Allows

# Approach

## Incorporate Cavitation Model in a Sequence of Platforms

- Potential Flow Model (Panel)
- Euler Analysis
- Navier-Stokes Analysis
- Navier-Stokes + Energy
  - Thermal Effects

# Governing Equations

- Navier-Stokes + Energy

$$\Gamma^{-1} \frac{\partial Q}{\partial \tau} + \frac{\partial (E - E_\nu)}{\partial \xi} + \frac{\partial (F - F_\nu)}{\partial \eta} = H$$

- Incompressible, Variable Properties

$$Q = \begin{pmatrix} p \\ u \\ v \\ T \end{pmatrix}, \quad E = \begin{pmatrix} \rho u \\ \rho uu + p \\ \rho uv \\ \rho vv + p \end{pmatrix}, \quad F = \begin{pmatrix} \rho v \\ \rho uv \\ \rho vv + p \\ \rho vT \end{pmatrix}, \quad H = \begin{pmatrix} 0 \\ 0 \\ 0 \\ q \end{pmatrix}$$

# Numerical Scheme

- 4-Stage Runge-Kutta Explicit Time-marching.
- Central Differencing in Space
- Local Time stepping
- Fourth order artificial viscosity used to prevent odd-even splitting.

# Euler Analysis

- Extend Classical Potential Flow Model to Euler Eqns.
  - Cavity Treated as a Constant Pressure Region
- Solve Direct Problem
- Specify Cavitation Pressure
  - Determine Inception Point and Cavity Length as Part of Solution
  - Use Closure Condition From Potential Methods

# Navier-Stokes Analysis

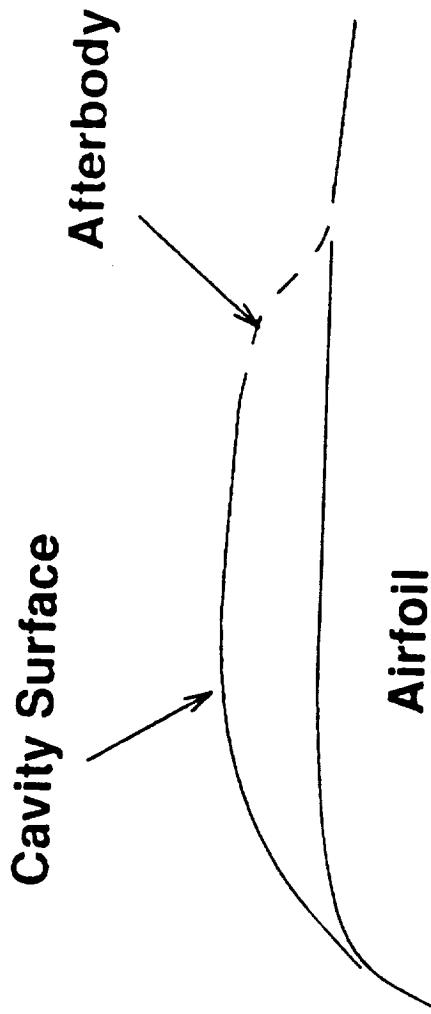
- Implementation Analogous to Euler Analysis
- Advantages
  - Enables the solution of viscous, turbulent flows
  - Couples thermodynamics through energy equation
  - Important for Cryogenics

# Cavitation Boundary Conditions

- Over-specified Boundary Conditions
  - Location of Solid Surface
  - Cavitation Pressure
- Cavity interface Location
  - "Linear" - B.C.'s transferred to Body Surface
  - "NonLinear" - B.C.'s applied on Interface  
Computational Domain evolves with solution

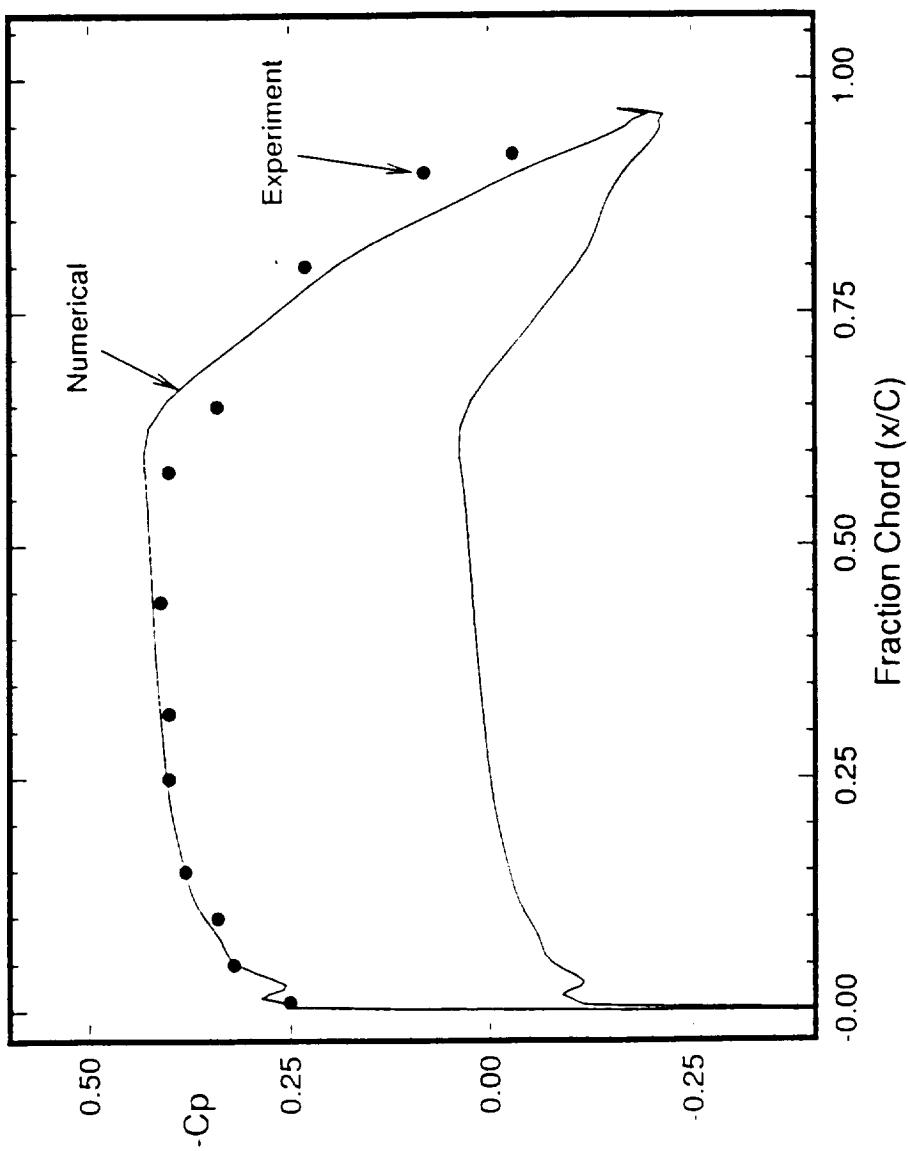
# Computational Steps

- Check pressure to identify cavitating points
- Trace cavity and ensure positive thickness
- Attach afterbody for finite thickness cavity
- Regrid domain (Nonlinear)



# Code Validation

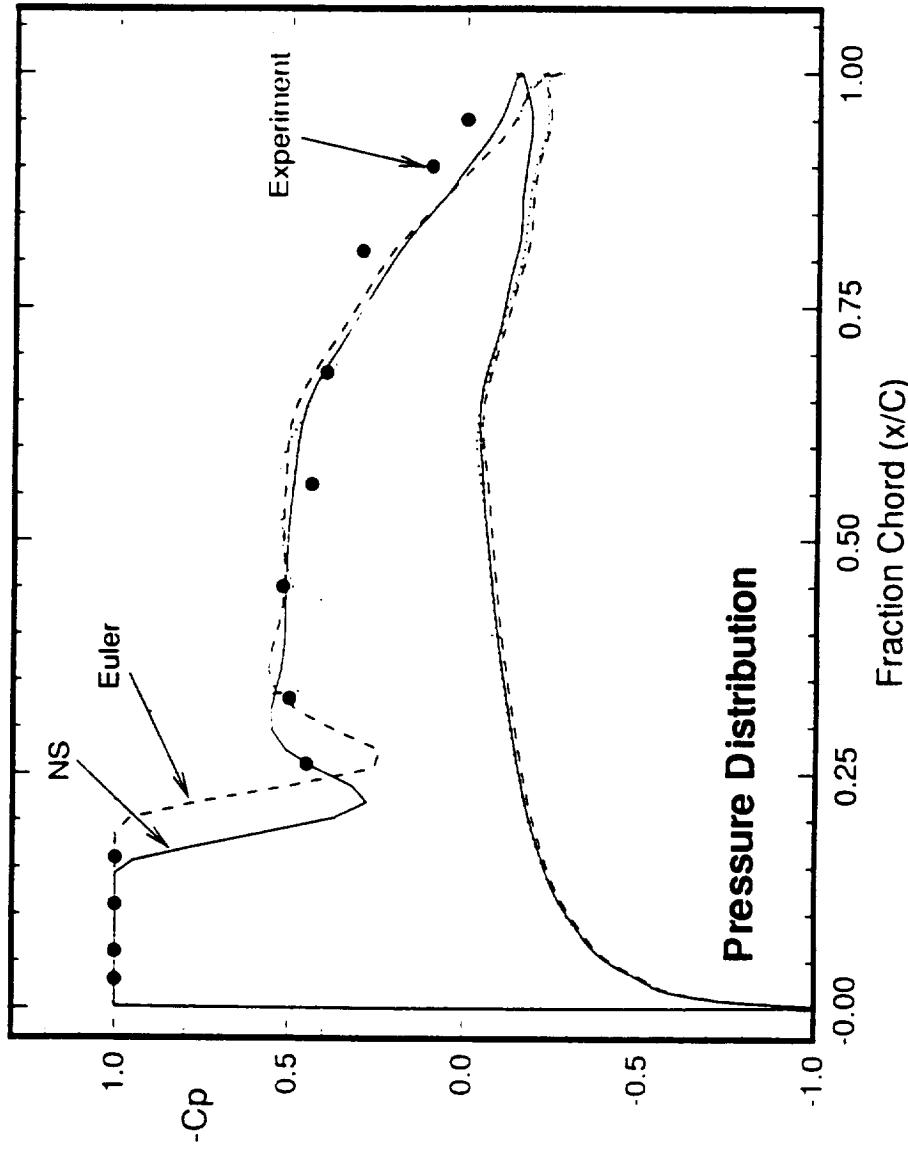
## Non-cavitating NACA66(MOD) Shen and Dimotakis



# Navier-Stokes/Euler Comparison

NACA66 (MOD)

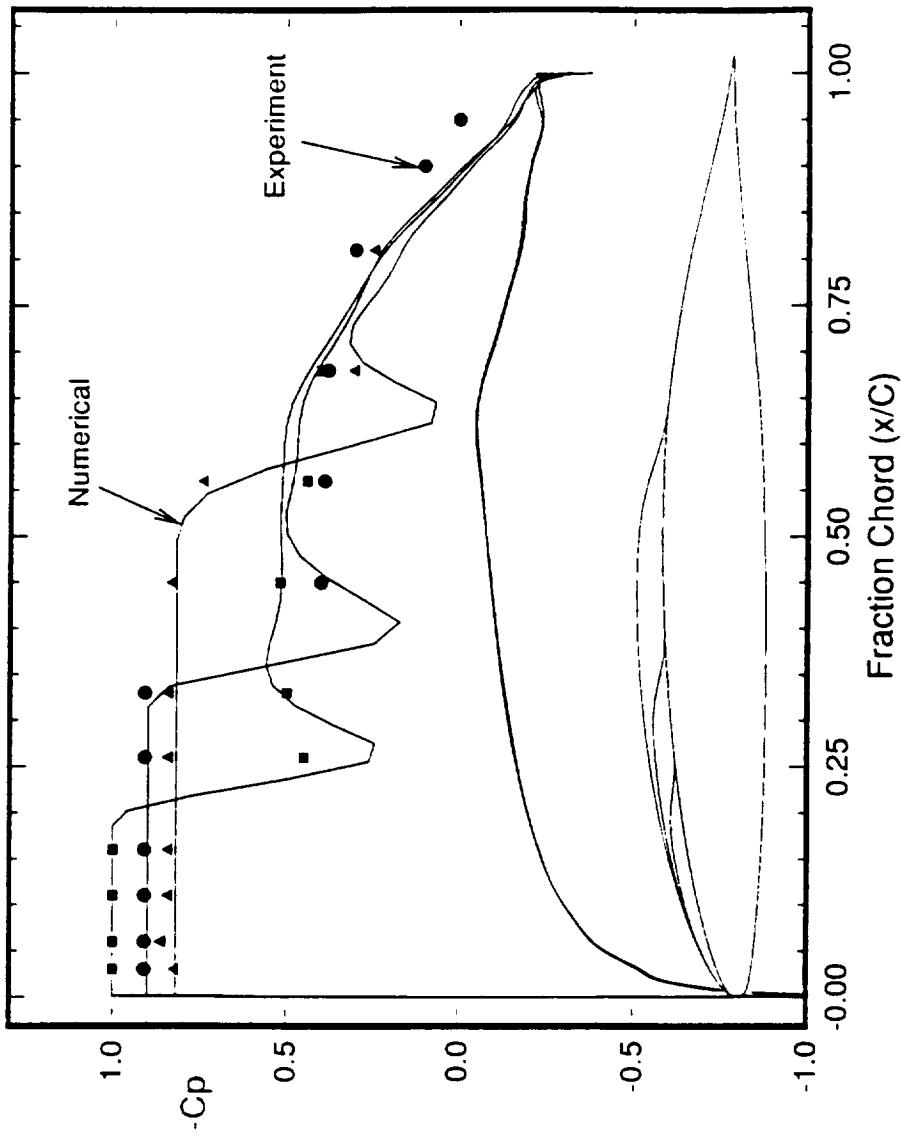
$\alpha = 4^\circ$



# Cavitation Pressure Comparison

NACA66(MOD) - Shen and Dimotakis.

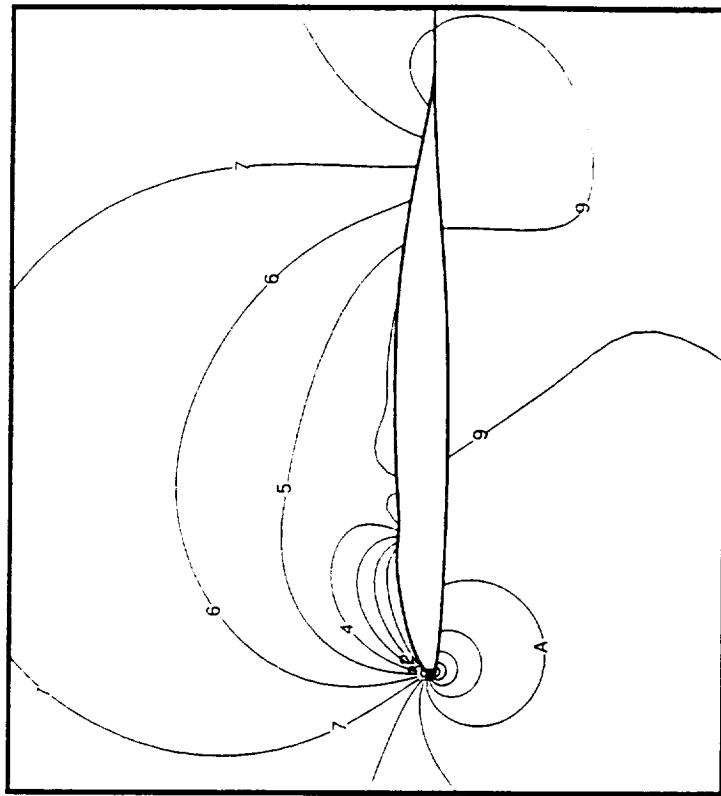
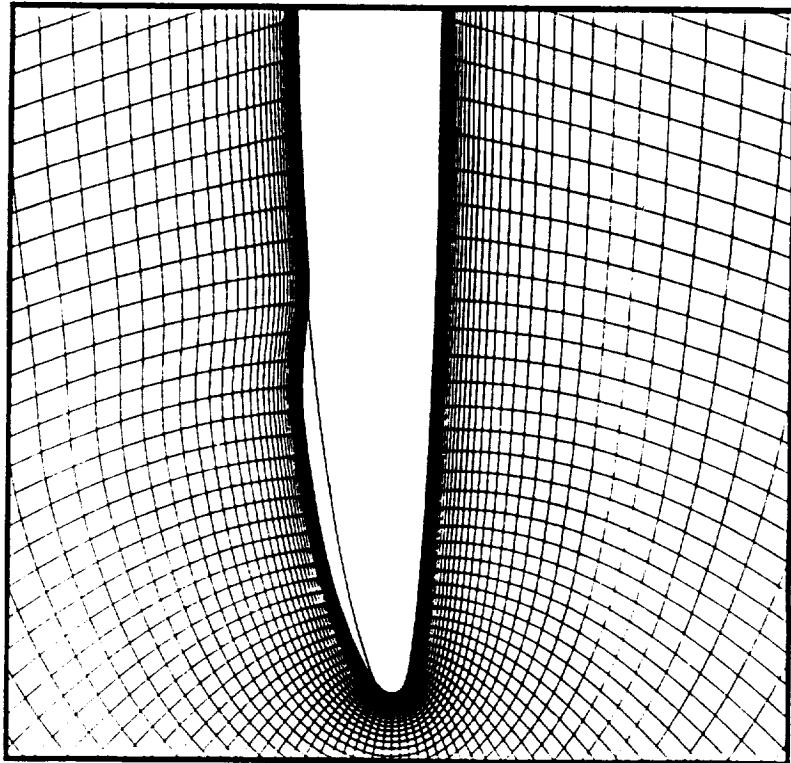
Pressure Distribution  
 $\alpha = 4^\circ$



# Cavitating Flowfield

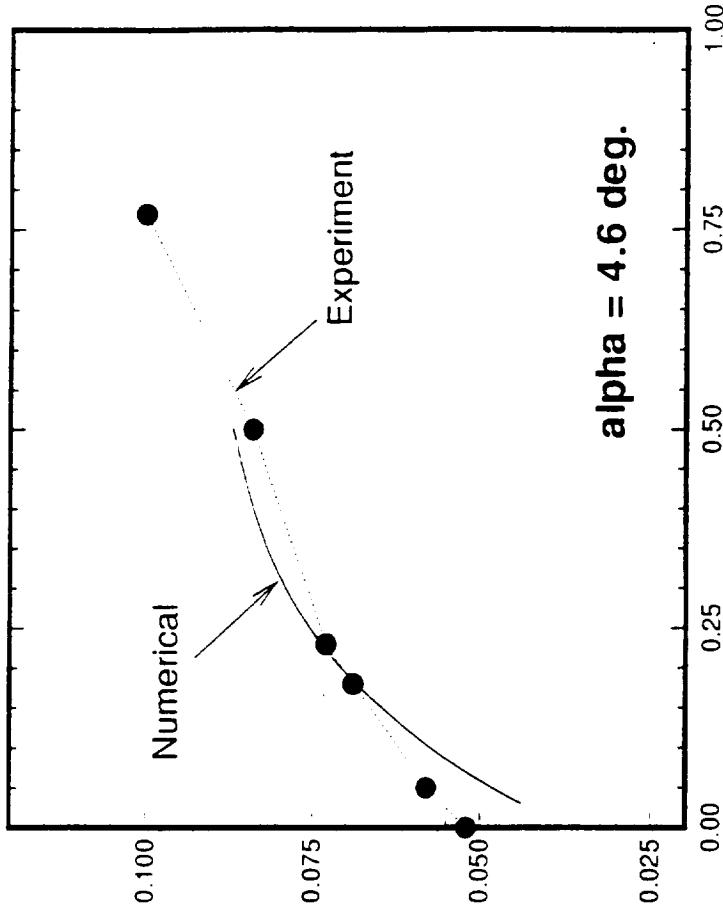
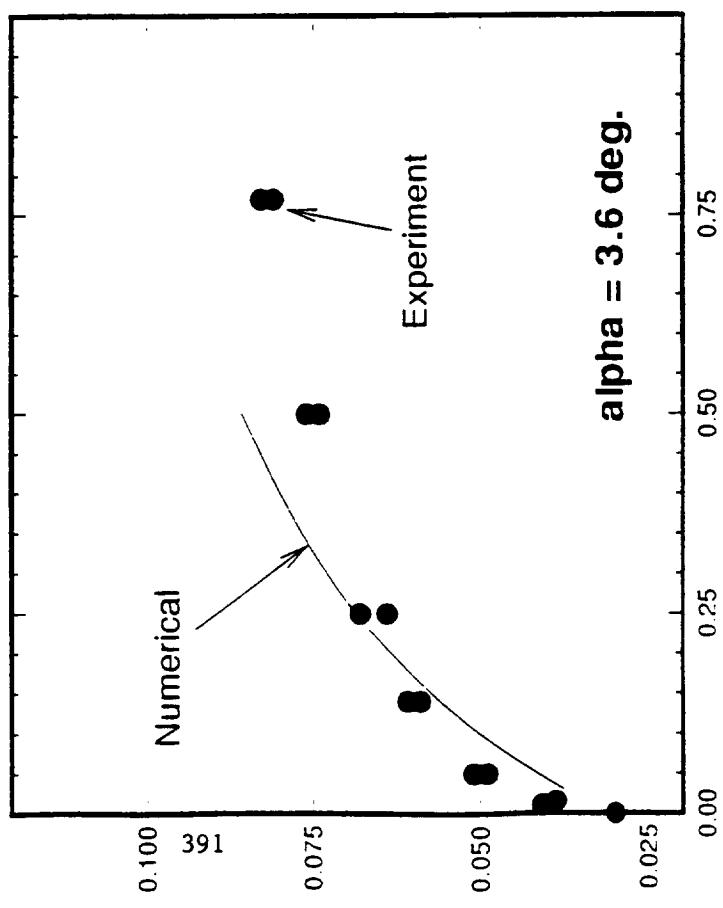
NACA66 airfoil

Final Grid



# Cavity Length Comparison

NACA 16009 hydrofoil - Dong (1983)  
 $\alpha/\sigma$  vs  $l/c$



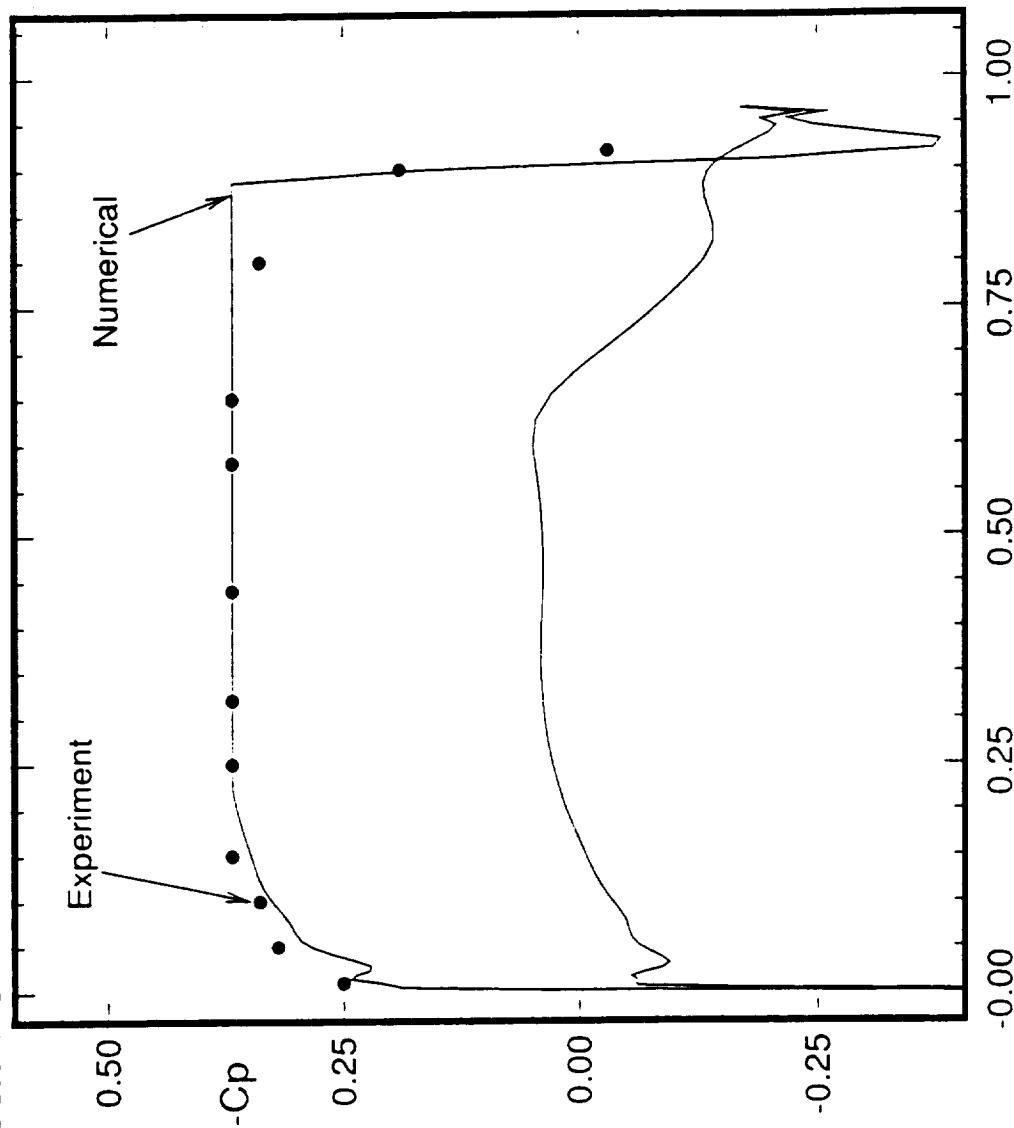
# Midchord Cavitation

- Midchord Cavitation occurs on blades with Flat Pressure Distributions
- Difficult to predict with potential flow codes
  - No distinct Minimum Pressure Location.
- Good Prediction using Euler Analysis
  - Specification of Inception Point not required

# Midchord Cavitation

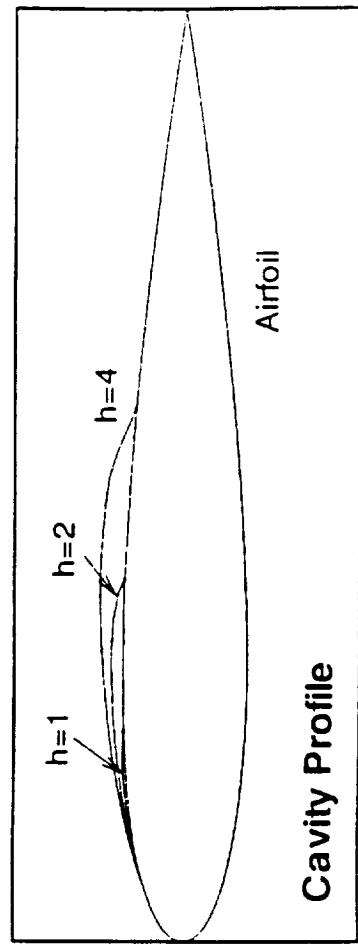
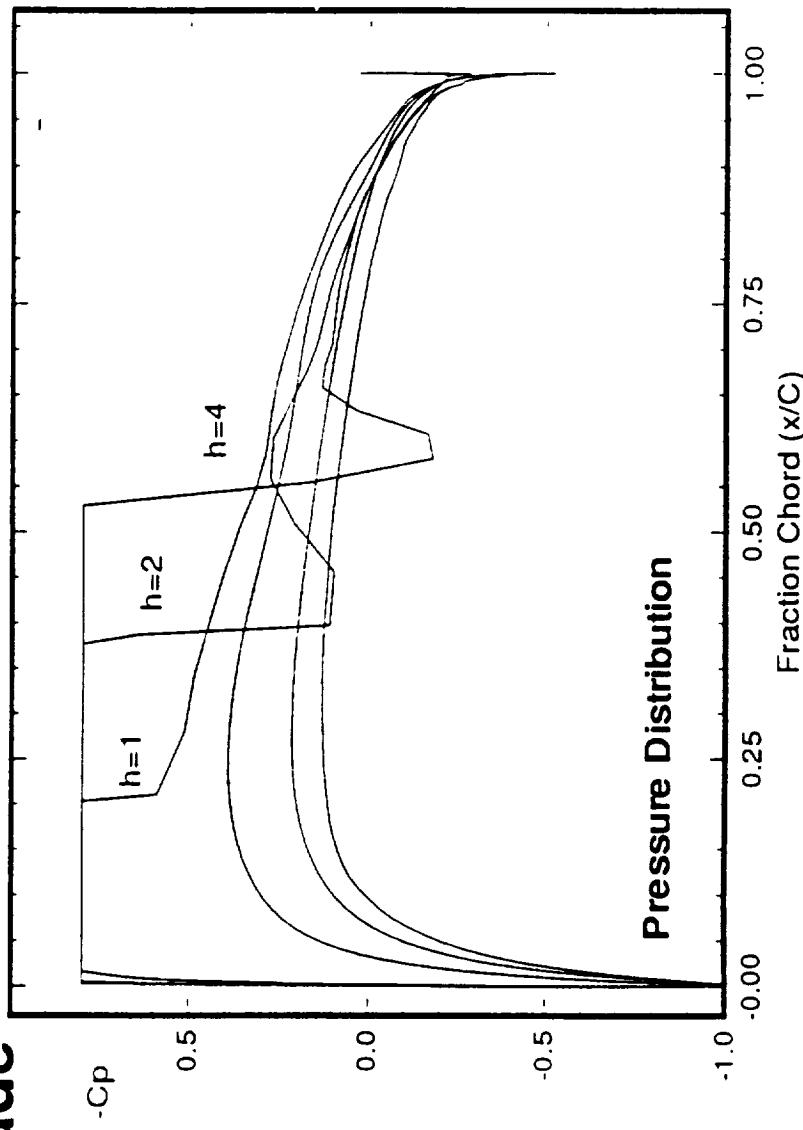
NACA66(MOD) - Shen and Dimotakis

Pressure distribution  
 $\alpha \stackrel{0}{=} 1$



# Effect of Cascade Spacing

NACA0012 Cascade  
 $\sigma = -0.5$



# Thermodynamic Boundary Condition

- Energy Balance at Cavity interface

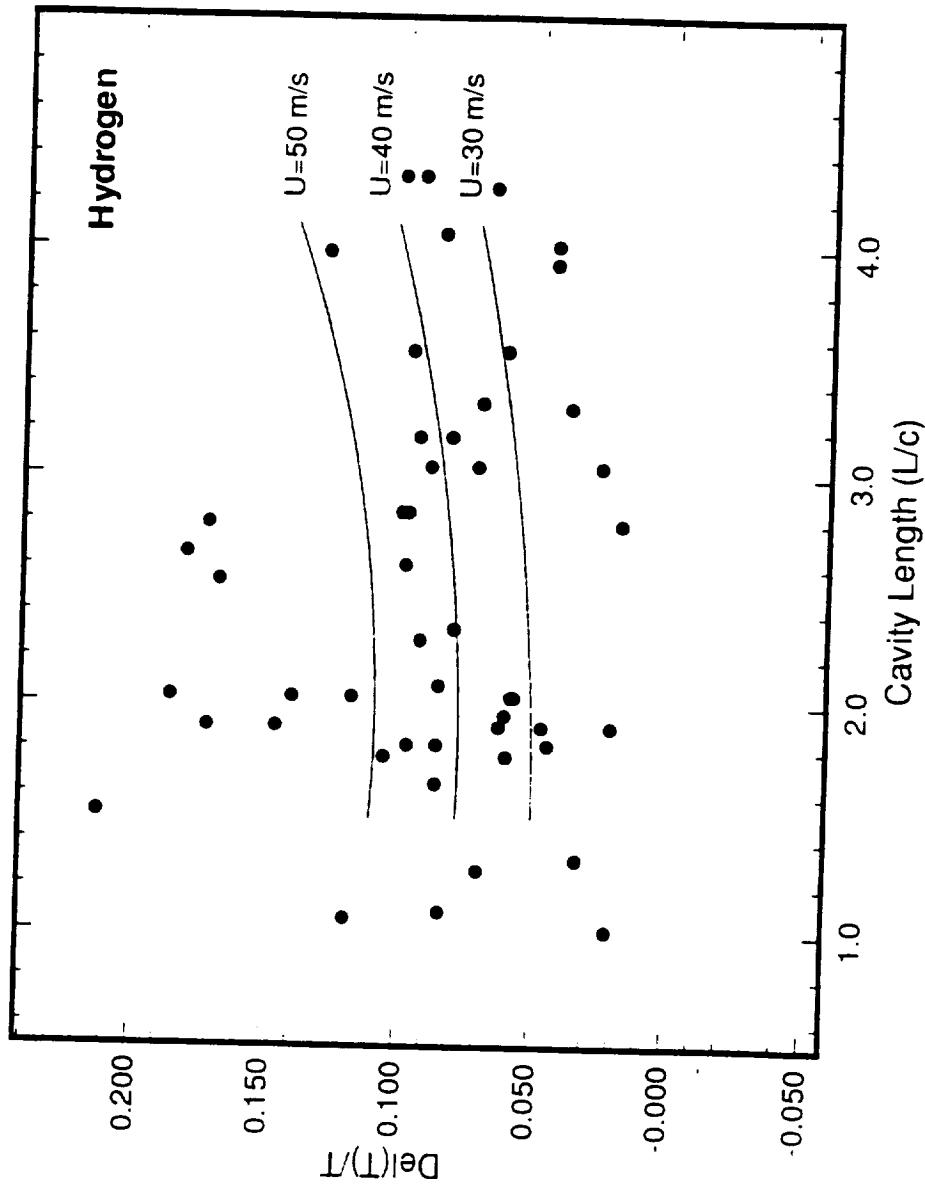
$$\text{Heat Conducted} = \frac{\text{Vaporization Rate} \times \text{Latent Heat}}{\text{into interface}}$$

- Determines the normal temperature gradient at surface
- Present Model Relates Vaporization Rate to Vapor Velocity

$$-u_c = u_\infty$$

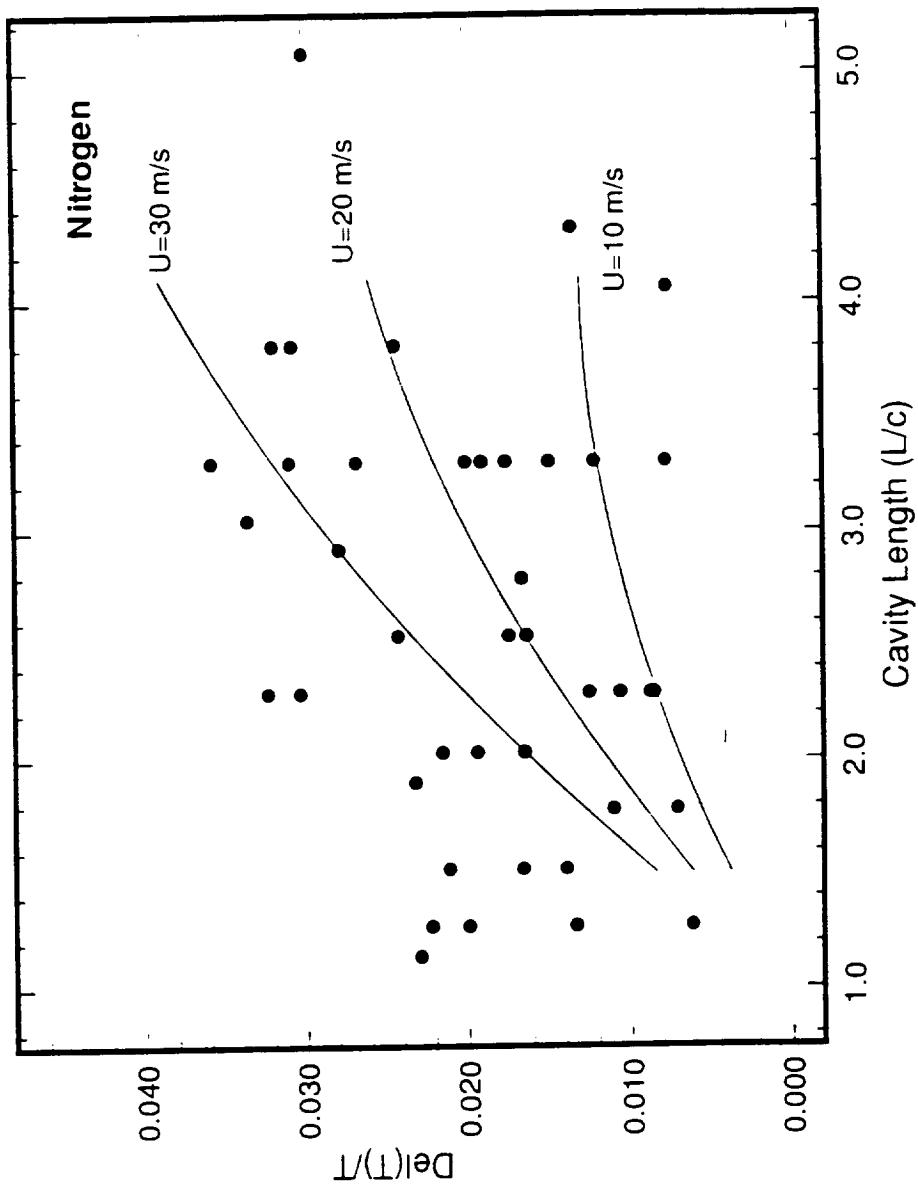
# Temperature Depression - Hydrogen

Hord (NASA CR-2156)



# Temperature Depression - Nitrogen

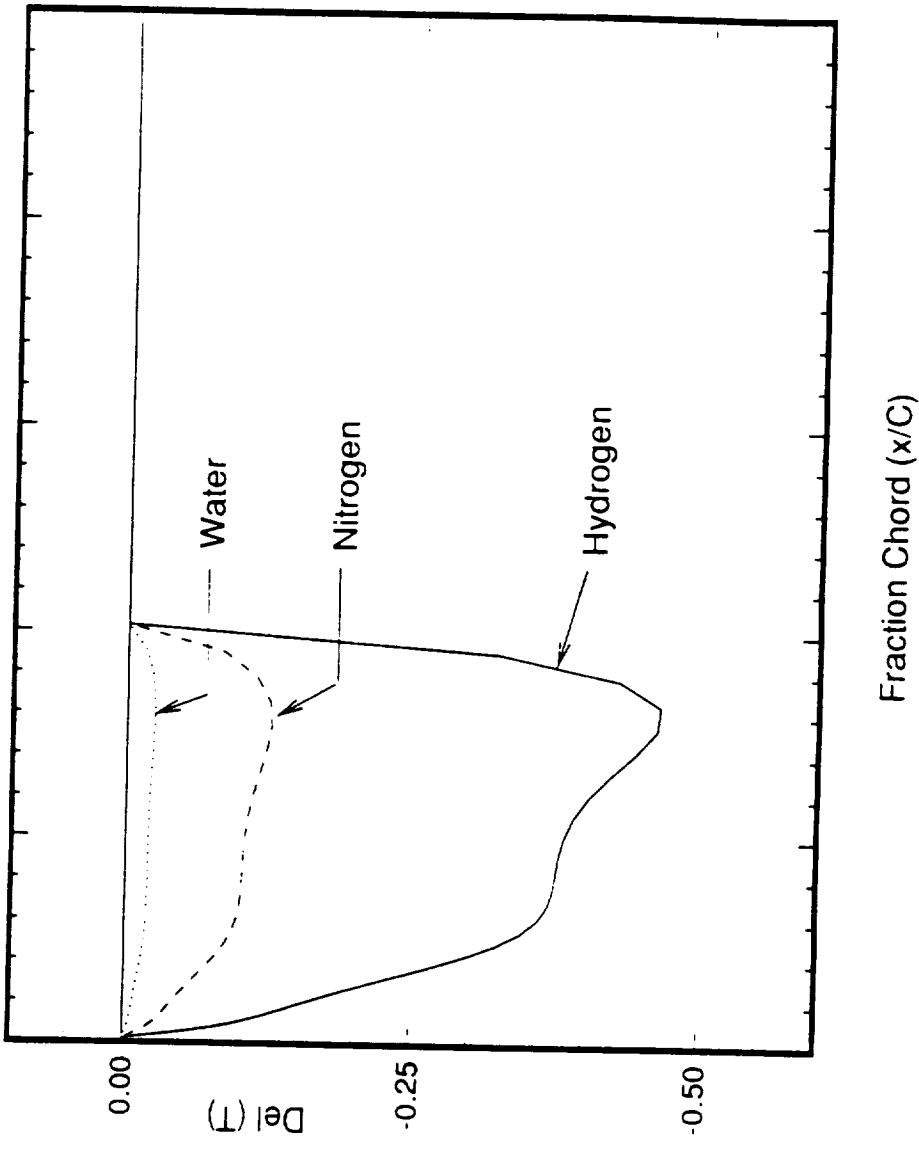
Hord (NASA CR-2156)



# Temperature Depressions

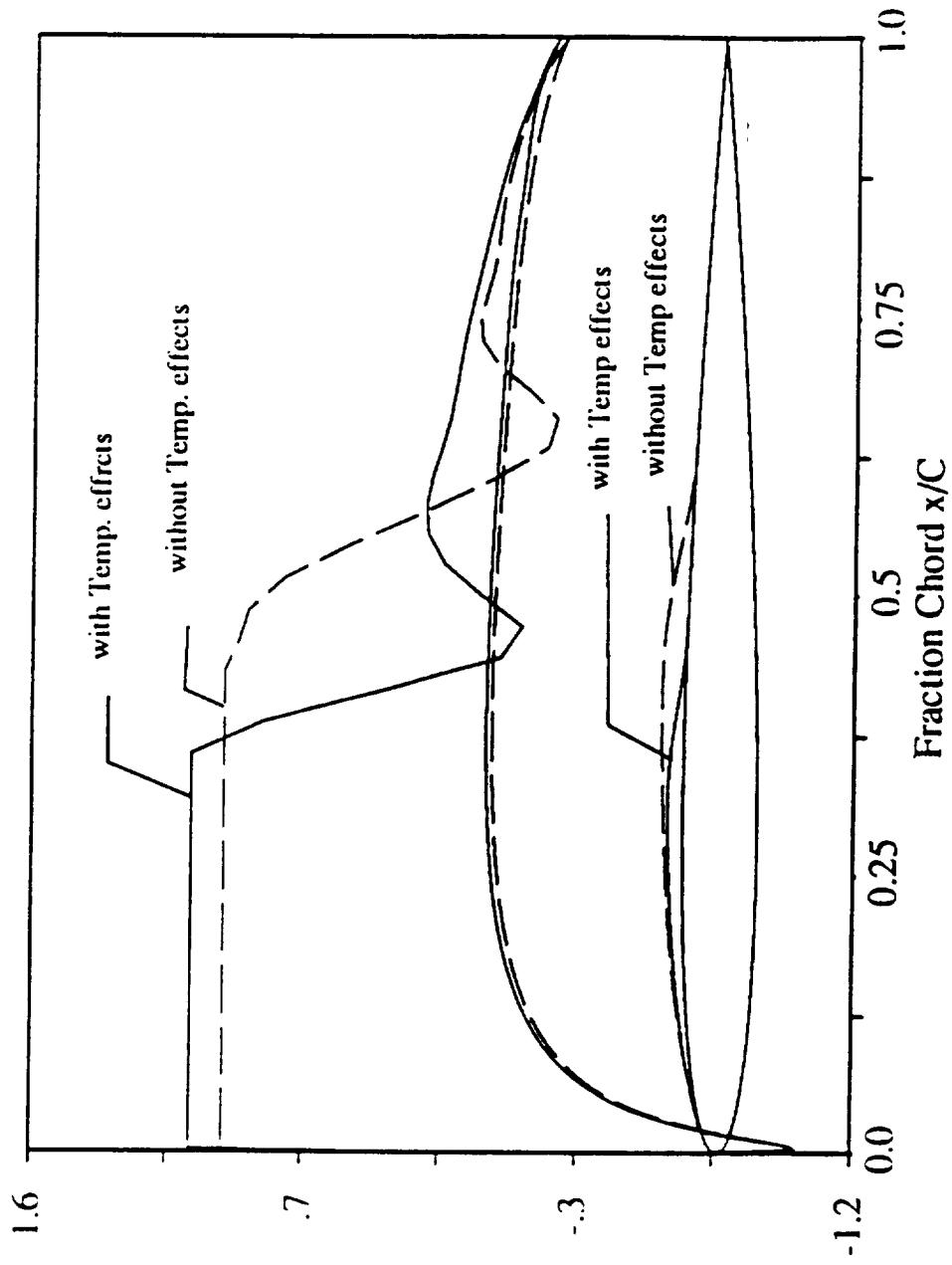
## Water, Hydrogen and Nitrogen

NACA0012 airfoil  
 $\alpha = 5^\circ$   
 $\sigma = -1.0$



# Impact of Thermal Effects on Cavitation

NACA0012 airfoil  
 $\alpha = 5^\circ$



# Summary

- Determines cavity length and inception point
- Good prediction of both pressure distribution and cavity geometry
- Capable of predicting midchord cavitation
- Cavity termination model used to predict pressure recovery.
- Easy to extend to more complex flows as well as to incorporate into design codes.
- Predictions of Panel, Euler and N-S are similar
  - BL's and Turbulence have little effect on Model Predictions

# Summary

- Thermal Effects are significant in cryogenic fluids
  - Slope of Vapor Pressure/Temperature curve is steep
- Near Super-critical Properties intensify Thermal Effect
  - Measurable Temperature Depression in liquid
- Effect requires coupled NS/Energy Solution
  - Thermal BL in Liquid
- Predict Temperature Depression
  - Empirical model for vapor production rate
- Model predicts magnitude of temperature depression satisfactorily

